where $a_1 = Kx = s$; in the present case, it is the radius of the semicircle in the physical plane, and a is the radius in the transformed plane.

Remarks

It may be remarked here in this connection that, in the Brown and Michael work,[3] the condition of zero net force configuration has been satisfied in the physical plane (i.e., the delta-wing plane). The transformed plane, i.e., θ plane, has been brought in the picture to facilitate the formulation of the potential flow corresponding to the physical plane. However, in the present problem, the flow investigation has been carried out in the transformed plane because the condition of zero net force configuration of the vortex sheet and the concentrated vortex can be satisfied in the transformed plane, since, in this plane also, the flow separates from the surface, and the vortex sheet is formed from the feeding points on either side of the body. It is true that, depending upon the geometrical shape of the body in the crossflow plane, the geometrical shape of the locus of the vortex center will also change; this is being reflected in some of the analytical derivations in the transformed as well as in the physical plane. One can easily write down the analytical relations in the physical plane in the following way. In place of Eq. (1), we start with the equation

$$\left(\frac{\tau - ae^{-i\pi/6}}{\tau + ae^{2'\pi/6}}\right) = \left(\frac{Z - [3(3)^{1/2}/4]a}{Z + [3(3)^{1/2}/2]a}\right)^{2/3} \tag{10}$$

from which we put

$$\tau/a = f(Z/a) \tag{11}$$

where

$$f\left(\frac{Z}{a}\right) = \frac{(3)^{1/2}}{2} \times \frac{\{1 + [3(3)^{1/2}/4](a/Z)\}^{2/3} + \{1 - [3(3)^{1/2}/4](a/Z)\}^{2/3}}{\{1 + [3(3)^{1/2}/4](a/Z)\}^{2/3} - \{1 - [3(3)^{1/2}/4](a/Z)\}^{2/3}} \tag{12}$$

The equation corresponding to Eq. (2) can be written as

$$F(Z/a) = -iU\alpha a(f - 1/f) - i\Gamma/2\pi \log(f - f_0) - i\Gamma/2\lambda \log(f + 1/f_0) + i\Gamma/2\lambda \log(f + \bar{f}_0) + i\Gamma/2\lambda \log(f - 1/\bar{f}_0) \tag{13}$$

The lift function can be written down in the following way:

$$L(x) = \text{the sectional lift in the crossflow plane}$$

$$= -\rho U\, Re\left[\int_C F(Z)\, dZ\right] = 21\ \pi\rho U^2 a^2 \alpha + \rho U \Gamma a \frac{(f_0 + \bar{f}_0)(f_0\bar{f}_0 - 1)}{f_0\bar{f}_0}$$

or, expressing the lift coefficient C_L as defined earlier, we find that

$$C_L = 6.23\left(\frac{\alpha}{\epsilon_1}\right) + 4.741 \frac{\Gamma}{2\pi a U \epsilon_1} \frac{(f_0 + \bar{f}_0)(f_0\bar{f}_0 - 1)}{f_0\bar{f}_0}$$

However, the stagnation-point consideration in the physical plane (cf., Eq. 3) leads to the following equation:

$$\frac{\Gamma/2\pi a U\epsilon_1}{(\alpha/\epsilon_1)} = \frac{(f_1 - f_0)(f_1 + \bar{f}_0)(f_0 + \bar{f}_1)(\bar{f}_1 - \bar{f}_0)}{(f_0 + \bar{f}_0)(f_0\bar{f}_0 - 1)} \tag{14}$$

and therefore,

$$\frac{C_L}{\epsilon_1^2} = 6.23\left(\frac{\alpha}{\epsilon_1}\right) + 4.741\left(\frac{\alpha}{\epsilon_1}\right) \times \frac{(f_1 - f_0)(f_1 + \bar{f}_0)(f_0 + \bar{f}_1)(\bar{f}_1 - \bar{f}_0)}{f_0\bar{f}_0} \tag{15}$$

The condition of zero net force configuration in the physical plane now becomes

$$\left[-iU\alpha a\left(1 + \frac{1}{f_0^2}\right) - \frac{i\Gamma}{2\pi}\frac{1}{[f_0 + (1/f_0)]} + \frac{i\Gamma}{2\pi}\frac{1}{f_0 - (1/\bar{f}_0)} + \frac{i\Gamma}{2\pi}\frac{1}{f_0 + \bar{f}_0}\right] - \frac{i\Gamma}{4\pi}\frac{f_0''}{(f_0')^2} = U\epsilon_1\left(2\frac{Z_0}{a_1} - 1\right)\frac{1}{(df/dZ)_0} \tag{16}$$

In case $f''(Z/a)$ can be neglected (which is possible only when the vortex center is very close to the feeding point) Eq. (16) becomes equivalent to Eq. (4) if we can interpret $(df/dZ)_0$ as the magnification of the scale of transformation between the physical plane and the transformed plane.

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Torque on a Satellite Due to Gravity Gradient and Centrifugal Force

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In order to analyze the rotational dynamics of earth-orbiting satellites during the preliminary design of their attitude control systems, it is customary and usually necessary to determine the approximate values of gravity-gradient torques on the satellites. Very often these values are based strictly on the gradient in gravitational force across the distributed mass of the satellite. For the case of a symmetric satellite in a circular orbit, such values might typically be calculated using the expression given by Nidey.[1] When this expression [Eq. (18) of Ref. 1] is written in scalar form, the result is

$$T_g = (\tfrac{3}{2})\omega_0^2(I_s - I_t)\sin 2\beta' \tag{1}$$

where

- T_g = magnitude of the instantaneous gravity-gradient torque vector, ft-lb
- ω_0 = orbital revolution rate, rad/sec
- I_s = moment of inertia about the axis of symmetry, slugs-ft²
- I_t = transverse moment of inertia, slugs-ft²
- β' = angle from the symmetry axis to local vertical (when $I_s < I_t$) or the horizontal plane (when $I_s > I_t$), rad

The torque acts about the transverse body axis that lies instantaneously in the horizontal plane, and the torque direction is such as to decrease β'. There are several examples in the literature wherein expressions equivalent to Eq. (1) are used to derive values of gravity-gradient torques.[2–5] These results imply erroneously that their results give a

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complete indication of the torque that tends to orient a satellite axis of minimum moment of inertia along the local vertical.

Actually, there is another external torque due to the natural space environment, of the same form as Eq. (1), which also tends to orient an axis of minimum moment of inertia along local vertical. This torque is due to the gradient in centrifugal force on particles of the satellite caused by minute differences in the radial distances to the particles from the center of curvature of the orbital path. As given by Davis,[6] the value of the centrifugal-force-gradient torque is

$$T_c = (\tfrac{1}{2})\omega_0{}^2(I_s - I_t)\sin 2\beta'' \tag{2}$$

where β'' is the angle from the symmetry axis to either the plane of the orbit (when $I_s < I_t$) or a normal to the plane of the orbit (when $I_s > I_t$) in radians. The torque acts about the transverse body axis that lies instantaneously in the orbit plane, and the torque direction is such as to decrease β''. Some references include this torque as part of the total "gravity-gradient" torque.[7, 8]

The vector sum of the gravity-gradient torque and the torque due to centrifugal-force gradient is a total torque vector that also tends to decrease β' and β''. Both the magnitude and direction of the total torque vector vary as either β' or β'' varies, and the general expressions for the component magnitudes of the total torque are quite complex. Therefore, the succeeding derivations are limited to special cases and average values.

When the total torque vector is horizontal and in the orbit plane, its magnitude is

$$T_h = (\tfrac{4}{2})\omega_0{}^2(I_s - I_t)\sin 2\beta \tag{3}$$

where β is the angle from the symmetry axis to the orbit plane for either $I_s > I_t$ or $I_s < I_t$. This case occurs when the symmetry axis is in a vertical plane normal to the orbit plane, and it illustrates the direct addition of the gravity-gradient and centrifugal-force-gradient contributions. The result is the maximum possible torque for a given value of β. When the total torque vector is vertical, its magnitude is

$$T_v = (\tfrac{1}{2})\omega_0{}^2(I_s - I_t)\sin 2\beta \tag{4}$$

This occurs when the symmetry axis is horizontal, and it illustrates the case when only the centrifugal torque is effective. The result is the minimum possible torque for a given value of β. Between these two extremes, the total torque vector varies sinusoidally in magnitude as a function of symmetry-axis orientation.

To determine the average total torque value per orbit, for a fixed orientation of the symmetry axis, the two contributions are considered separately. The average gravity-gradient torque is given in Ref. 1 as

$$T_{g\,\mathrm{av}} = (\tfrac{3}{4})\omega_0{}^2(I_s - I_t)\sin 2\beta \tag{5}$$

where β' can be replaced by β because of the averaging. The instantaneous torque components normal to the orbit plane all cancel out over one orbit, and the resulting average torque vector is in the orbit plane. The average centrifugal-force-gradient torque is

$$T_{c\,\mathrm{av}} = (\tfrac{1}{2})\omega_0{}^2(I_s - I_t)\sin 2\beta \tag{6}$$

where β'' of Eq. (2) can be replaced here by β because of the averaging. The directions of the two average torque contributions of Eqs. (5) and (6) are the same, and thus they are added directly. Therefore, the average torque due to gravity gradient and centrifugal force is

$$T_{\mathrm{av}} = (\tfrac{5}{4})\omega_0{}^2(I_s - I_t)\sin 2\beta \tag{7}$$

When $I_s < I_t$, the torque direction is such as to decrease β to zero; when $I_s > I_t$, its direction is such as to increase β to 90°. Thus, Eq. (7) represents exactly the same type of moment as Eq. (5) [Eq. (23) of Ref. 1]. However, an improvement has been made by the increase in magnitude due to the centrifugal-force gradient. Therefore, Eq. (7) should be used for the determination of average "gravity-gradient" torques in preference to other expressions that do not account for the gradient in centrifugal force.

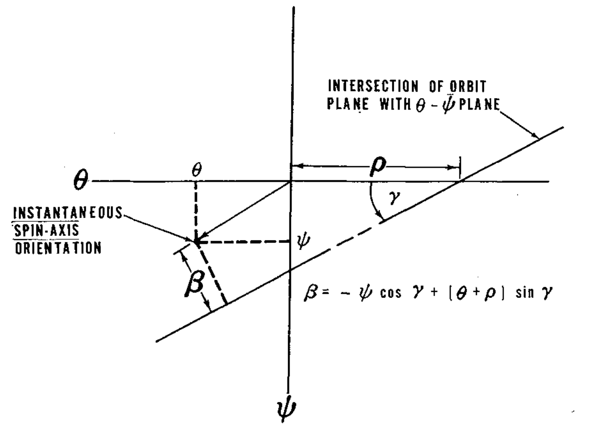

Fig. 1 Orientation of spin axis in θ-ψ plane and its angle β from orbit plane.

Probably the most useful application of average values of gravity-gradient and centrifugal-force-gradient torques is in the dynamic analysis of spinning satellites. Since the spin axis maintains a relatively fixed orientation in inertial space, the value of β in Eq. (7) is constant over the time period of one orbit for which the average value is taken. Therefore, if the torque of Eq. (7) can be resolved into torque components acting about the satellite body axes, the equations of motion for precession due to gravity-gradient and centrifugal force can be written.

For this purpose, it is first assumed that the spin axis does not drift far from the orbit plane, so that β is small. Thus $\sin 2\beta = 2\beta$, and Eq. (7) is linearized. Next, β is expressed in terms of the Euler angles of the spin axis from its initial orientation in inertial space. This can be done by observing the spin-axis orientation in the θ-ψ plane relative to the orbit plane. The angles θ and ψ are the Euler angles representing the deviation of the spin axis from its initial orientation. In Fig. 1 the orbit plane is shown as its intersection with the θ-ψ plane, which is normal to the initial spin axis orientation, and the angles ρ and γ represent the orbit-plane orientation. By a geometrical construction in Fig. 1, it can be shown that

$$\beta = -\psi\cos\gamma + (\theta + \rho)\sin\gamma \tag{8}$$

Thus, by substitution into Eq. (7), the total torque on a spinning satellite due to gravity-gradient and centrifugal force is given as

$$T = (\tfrac{5}{2})\omega_0{}^2(I_s - I_t)[-\psi\cos\gamma + (\theta + \rho)\sin\gamma] \tag{9}$$

The torque vector is in the orbit plane, and it can be resolved into satellite transverse body axes by a simple transformation through the rapidly changing Euler angle ϕ. Since $\phi = \omega_s t$, the body-axis torques vary sinusoidally at the spin rate. Thus,

$$T_y = -(\tfrac{5}{2})\omega_0{}^2(I_s - I_t)[-\psi\cos\gamma + (\theta + \rho)\sin\gamma] \times \sin(\gamma - \omega_s t) \tag{10}$$

$$T_z = (\tfrac{5}{2})\omega_0{}^2(I_s - I_t)[-\psi\cos\gamma + (\theta + \rho) \times \sin\gamma]\cos(\gamma - \omega_s t) \tag{11}$$

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Modification of the Hydrazine-Nitrogen Tetroxide Ignition Delay

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IGNITION delay measurements were made on the system hydrazine-nitrogen tetroxide, and the effect of additives on the ignition delay time was determined. The apparatus used was of the type developed by Kilpatrick and Baker,[1] but was modified as described below. Delay times were both shortened and lengthened by additives, but the majority of the compounds tested tended to shorten the ignition time.

Vapor-phase studies of the hydrazine-nitrogen tetroxide system have been reported in the literature,[2] and a thermal rather than free radical reaction mechanism has been proposed for the reaction. Studies of liquid phase interactions of hydrazine and nitrogen tetroxide have been concerned primarily with explosion hazards,[3] hence, ignition delay times were not reported. The purpose of the experiments reported here was to determine the effect of additives on the ignition delay time for liquid nitrogen tetroxide and hydrazine. Additives were selected with the hope that a clue to the reaction mechanism could be obtained from the delay data. For example, thermal moderators, free radical traps, free radical sources, and surface active agents were used. The latter agents were used because of suspected immiscibility of the two reactants. The fact that nitrogen tetroxide and hydrazine are immiscible was subsequently proved by photographing[4] the dropwise addition of N_2O_4 to N_2H_4.

Apparatus and Experimental Procedures

The apparatus for the measurement of ignition delay times consisted of a reaction chamber of approximately 450-cm³ volume, two injection pistons of 0.515-and 0.466-in. diam and 0.375-in. stroke, and a driving piston that, when acted upon by compressed gas, drove the two small pistons simultaneously. The drive gas was contained in an accumulator bottle of about 1-ft³ volume at 1500 psig and was released by a quick acting solenoid valve (90% open in 2 msec).

Hydrazine was contained behind the larger of the two injection pistons; thus, all measurements were made at a volume ratio $N_2H_4/N_2O_4 = 1.22$. Valves located in the casing surrounding the small pistons facilitated refilling between runs. An overflow was incorporated into the fill system to insure the absence of gas bubbles. The propellant storage spaces were sealed from the reaction chamber by 5-mil teflon disks, which were held in place by washers and lock bolts. Each of the lock bolts was drilled with a 0.060-in hole that constituted the injection orifice. The two streams were injected at 90° to each other in a swirl chamber, resulting in tangential mixing. The reaction chamber was cleaned after each run and was flushed with nitrogen for 20 min prior to each measurement.

Delay time was measured by photographing an oscilloscope trace that recorded both light emission and pressure. The photocell pickup was located in the end of the reactor to insure the maximum view of the reaction chamber. The photocell amplifier was operated at maximum gain to insure the detection of the initial light release. The pressure sensor was mounted flush with the inner wall of the reactor and was located 3 in. from the injection ports.

In initial runs, a 1-mil teflon seal was used between propellants and reaction chamber. It was observed in these runs that light was emitted coincident with a small pressure rise and that the more rapid pressure rise conventionally associated with ignition occurred later. It was also difficult to obtain reproducible results in these experiments. This initial small pressure rise was identified as ignition by N_2O_4 vapor, which had prematurely leaked by the teflon disk and

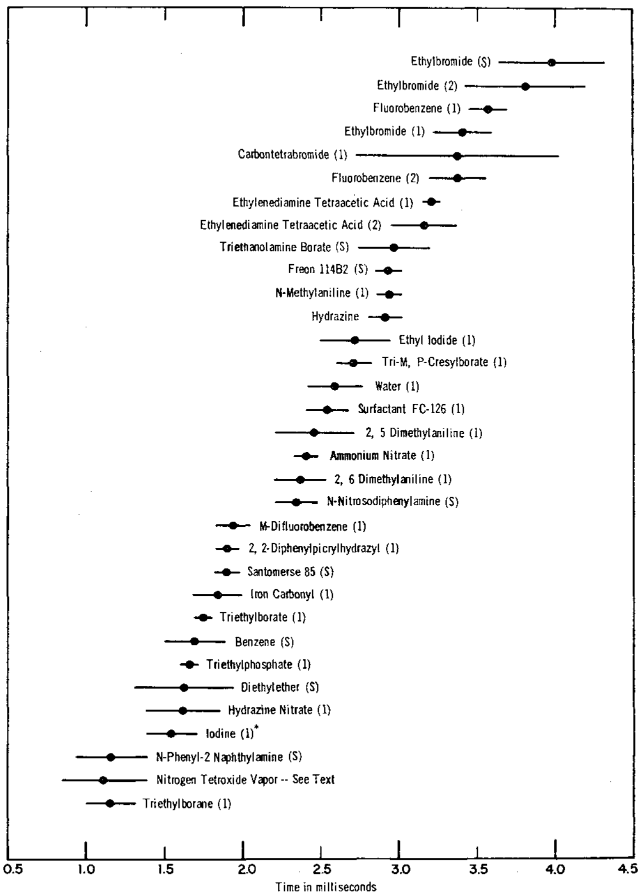

Fig. 1 Ignition delay data where an asterisk indicates that the substance is added to N_2O_4.

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